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CRITICISMS AND DISCUSSIONS.

LOGIC AND PSYCHOLOGY.

The nature and purpose of symbolic or mathematical logic, which began to be developed by Leibniz and was continued quite independently by Boole and others, is tolerably well known by now. Logical reasoning is translated by it into what Leibniz called a "real characteristic" which is very analogous to ordinary algebra, and helps swiftness and accuracy of reasoning—even complicated reasoning—in much the same way as the signs in algebra do. This tendency culminated in the very ingenious and useful "mathematical logic" of Peano. Peano's system was far more complete than Boole's, for the whole of a piece of reasoning which included algebraic formulas and equations could be put into a symbolical form in which ordinary words—which are not part of a "real characteristic"—are not used. In this direction Peano's system met the much earlier system devised by Frege. However, Frege's system was not thought out so much with a view to rapidity of reasoning and convenience of writing as with a view to emphasizing slight and important logical distinctions in very similar concepts and deductions and consequently a scrupulous accuracy in deductions. It may thus be noticed, by the way, that the purpose of Frege's symbolism was different from that of all previous symbolisms in logic and mathematics, for Frege wished to lay stress upon the *differences* in various analogous ideas and deductions rather than upon their *analogies.* Broadly speaking, Russell and Whitehead's work may be characterized by saying that it is formed under the influence of a combination of the two tendencies represented by Frege and Peano. The convenient symbolism of Peano is retained wherever possible and the superior analysis and subtlety of Frege is fully used. We ought to add also that nearly all of Frege's discoveries were made independently by Russell himself, Frege's great work having been neglected by philosophers and mathematicians.

There is one great point in which Russell's works differs much from that of Frege: full use is made of the enormously important researches of Georg Cantor on transfinite numbers. While putting on a firm basis the treatment of infinite classes and numbers, Cantor's work led to the recognition of forms of a paradox absolutely fundamental in logic. After many vain attempts by various mathematicians and philosophers, this paradox has been satisfactorily solved by the thorough remoulding of logic given in Whitehead and Russell's *Principia Mathematica.*

From Peano's various *Formulaires* to the work last mentioned the subject-matter is principally the collection of truths which we can reach by logical deduction from logical principles. This body of truths is not a description of psychological methods of discovery or psychological results, but is of course reached by psychical processes, like most other discoveries in a purely intellectual domain. It is then simply irrelevant to complain that there is no place in the *Formulaires* or *Principia* for that "intuition" which brings about mathematical discoveries. It would be just as much to the point to complain that in what is excavated we do not discover the tools used for excavating or the method of excavation. And yet this is what the rather superficial and amusing discussions of Henri Poincaré are mostly about. And these discussions are what Prof. J. B. Shaw in the number of *The Monist* for July, 1916, refers to (p. 397) as Poincaré's "successful attacks on logistic." We might reasonably, it seems to me, have expected that Professor Shaw should make some reference to the reply by Louis Couturat to Poincaré which was translated in *The Monist* for October, 1912, and which is quite conclusive on so many points. Professor Shaw, in his eloquent and somewhat inaccurate (both from the points of view of history and logic) attack on mathematical logic, urges what are, at bottom, the very same irrelevant arguments. I shall try to point out some of these inaccuracies, both because they are fairly common even now among mathematicians, and because it is surely the duty of every one to contribute as far as he can to the clarification of notions in America above all other countries; for it is from America that we expect an exceedingly large proportion of the work of the intellect in future now that Europe has deliberately handicapped herself.

Professor Shaw's slighting remark on the impotence and boasting power of logistic (p. 411) is the result of a strange misconception. Logistic deals with logical entities and deductions which are fundamental to mathematics, and it is unjust to try to make people believe that logistic ever claimed to be the overlord of mathematics. There seems, in fact, to be a note almost of personal dislike for logistic in those mathematicians who attack it. And yet the question is wholly concerned with logical facts, and is not to be answered by rhetorical appeals to prejudice or sentiment. If logic is more fundamental than mathematics, why should there be any objection to the—successful as it happens—attempt to define mathematical entities in terms of logical ones? If mathematics is more fundamental than logic, the first thing to do is to draw up a scheme showing that logical entities can be deduced from specifically mathematical ones. Until this is done, and certain objections to it are at once obvious, it is quite unconvincing to disparage cultivators of logistic. After all, logisticians are working at mathematics in much the same way that other mathematicians are. They are concerned with more fundamental problems and problems which do not so easily appeal to the public, as, say, a proof of Fermat's great theorem would, but they discover truths just as much as any other mathematicians. They introduce conceptions to work with. We may mention the idea of *propositional function* actually mentioned by Professor Shaw in terms of commendation (p. 411), which was introduced implicitly by Boole and MacColl—both early mathematical logicians — and explicitly by Frege, Peano and Russell — all logisticians. A small acquaintance with such a work as that of Frege will give plenty of examples of other powerful new ideas introduced. And then as to truths discovered by logisticians, we may remind Professor Shaw that the solution of "the paradoxes of logic" is wholly due to them, while mathematicians who were unacquainted with logistic hopelessly floundered in the search for a solution. Twelve years ago I was one of these flounderers myself, and my "solution" had been accepted as satisfactory by many mathematicians.

The real fact is that these results of logistic do not strike some mathematicians as nearly so important as some of the results of the theory of functions, for instance. I think they forget that it is only in virtue of all truths being really of equal "nobility" that Jacobi was right in claiming that a theorem in the theory of numbers was just as fine as a very striking result in mathematical astronomy.

Perhaps the greatest mistake made by Professor Shaw is the extraordinary statement about the nature of truth near the top of page 409. It is surely quite evident that truths themselves do not develop. That twice two are four was just as true last year as it will be next year,—even if no people at all are left alive on the earth next year. Professor Shaw finds fault with something I wrote because he thinks that I maintained that ideas are not created by man. It is quite evident from what I said in the context that I only held that *truths* are not created, though I certainly said in a slipshod and inaccurate way that "we do not really create *anything* in science." Really Professor Shaw shows afterward that he agrees with me that truth itself is not created, and his remark that doubtless I thought that words and ideas waited in the mines of thought for the lucky prospector does not appear to be either logical or a good guess (see pp. 409-411). However, at the top of page 409 he remarks that the world of universals changes in time. I suppose that he means that our ideas, say of an "integral" or "continuity" have changed; but I hardly think that he ought to have fallen into the error of mistaking the thing itself for a result of our groping after the thing. I take it also that he does not intend to say that truth evolves, for that rests on a confusion between a *proposition* and a *propositional function,* such as in thinking that such a function as "Dr. Wilson is President of the United States" is a proposition and not a function of the time which becomes a proposition when any instant is specified and is then constantly true or false eternally. What is the case seems to me to be that in logic and mathematics the world we are concerned with is a world of facts, not of conceptions. *Conceptions* are formed by us for the purpose of stating truths, and in the world of pure mathematics we only come across facts and *form* and *variables.* In this I think that I shall have the support of one at least among philosophers: I refer to Dr. Carus, who has always maintained that mathematics is essentially concerned with the ideas of *form* and *"anyness."*

We now come to the last inaccuracy in Professor Shaw's paper that I shall deal with. This is the question about the logic of infinity. The inaccuracy of the statements on page 412 appears clearly if we give a short statement of the facts in the treatment of infinity by mathematicians and logicians. Georg Cantor, in a series of works dating from 1871 to 1897, succeeded in founding a new and immensely important theory of transfinite numbers. The use of the lowest transfinite cardinal numbers did not and does not present any difficulty whatever to mathematicians or even logicians; but, as Burali-Forti, Russell, and others noticed in various forms the *whole* series of transfinite numbers presents difficulties which were later found to be fundamental logical difficulties of the same nature as that of the Cretan who said that all Cretans were liars. Such problems were discussed at length in Russell's *Principles of Mathematics* of 1903 and in the years after the publication of this book were satisfactorily solved by him and Whitehead. These solutions may be found in the *Principia* of 1910, and in the almost wholly symbolical form of the book last mentioned it is naturally impossible, even if it were not superfluous, that the claims made in the earlier work should be repeated. Thus it is unjust to conclude (p. 404) that the *Principia* is an abandonment of the claims of the *Principles,* brought about because of the difficulties found in Cantor's work. One might just as well conclude that the difficulties of a solution of the great difficulty of "Cantorism" had made Russell give up joking, for there are many jokes in the *Principles* and only one in the *Principia.* There is one more point. It is only what we may call a "boundary problem" about Cantor's numbers that gives rise to difficulty: the resolve that any object about which we talk or reason must be defined in a finite number of words (p. 413) does not succeed in putting out of court all classes that have an infinite number of members. Infinite classes of objects each of which can be finitely defined can be defined in a finite number of words, or better symbols of a "real characteristic." The class of prime numbers is such a class. If indeed we may use the notion of *any* (which is represented by one word) or the notion of a *variable* in general, we cannot avoid admitting definitions of infinite classes by a definite number of words. If also we may use a sign for a variable, there is no earthly difficulty in giving a general rule for correspondence in a way that is denied by Professor Shaw on page 413. The rule, for example, if n is an integer, given in the formula $n+p$, where p is another integer, indicates precisely another class of integers which is correlated to the whole class of integers considered first.

There is a small logical error committed by Professor Shaw, at least if he considers philosophy to be the same thing as metaphysics, which may explain why he is so satisfied with himself for ignoring philosophy. On page 409 he characterizes a certain assumption as "philosophical" and explicitly divides "philosophy" from "mathematics." On page 414 he agrees with Lord Kelvin that "mathematics is the only true metaphysics." Thus he would seem to hold that there is no such thing as philosophy at all; this would certainly explain why philosophical assumptions are so little worth serious discussion.. Such discussion would in fact be as foolish a problem as to investigate the birthplace of Jack the Giant Killer's hen. But if we try to take a somewhat broader view, and are not satisfied with dividing our knowledge into arbitrary watertight compartments labeled "Philosophy," "Mathematics," and so on, we see that there are certain logical questions which can be and have been solved by symbolical methods which strongly remind us of algebra, which are absolutely fundamental in mathematics, and which when formulated in ordinary language sound so like what professional philosophers have often talked about that many are tempted to hurry them out of sight into the "philosophical" compartment. These are some of the questions with which logistic deals. Logistic never claimed to be able to run without the guidance of a human intellect (see p. 411) any more than the sciences of mathematics or logic or chemistry did. What it does claim to do is, like ordinary mathematics, to save our minds the labor of performing again each elementary reasoning which requires no talent but only memory—often a prodigious memory when the reasoning is complicated; so that we can reserve all the talents we may possess for overcoming those obstacles to a discovery of truth that have not been hitherto overcome. Then again, unlike ordinary mathematics, logistic seeks to point out differences in analogous ideas and reasonings which play an even greater part then analogies when we come to consider really subtle reasoning. Thus the analogy between implication between propositions, inclusion between classes, and inclusion between relations breaks down in certain cases, and we see that Russell in his later work forsakes the identical form of the symbols expressing these relations. Peano, as we know, kept to the same symbol on account of the very close analogy between the relations spoken of.

If we are content to accept without examination the arbitrary classification of people who were unacquainted with modern logic into exclusive "mathematical" and "philosophical" compartments, we must be prepared to think we see what we think are the rigid foundations of mathematics being eaten into by philosophy, and if we wish still to maintain that the foundations of mathematics are rigid we shall have continually to give the new name of "philosophy" to parts of what were hitherto considered to be mathematical. This state of things was actually brought about by what Poincaré called "Cantorism": truths which were hitherto considered solid and mathematical seemed to be thrown into doubt by the advance of philosophy. Of course this was not really so: the logical questions at the foundation of mathematics are capable of scientific investigation just as much as the theory of numbers of the differential calculus, and it is unnecessary and ridiculous to narrow the scope of our investigations because we shall meet logical difficulties if we do not. What would be thought of a tradesman who thought he could calm the mind of his assistants by maintaining that the ravages of a bull, although they seemed to be in his own china shop, were really in a drapery department which had somehow extended into that part of his shop where plates were sold? This is what those mathematicians do who dismiss awkwardness to "philosophy" and think that thereby they have kept mathematics pure and free from all "metaphysical" discussions.

Miss Dorothy Wrinch has sent the following comments on Professor Shaw's article in *The Monist* for July, 1916:

"The chief thing that I quarrel with in Professor Shaw's article is his idea of *one*: selecting one pencil from a pile is really rather different from considering the class whose members are the classes 'living kings of England,' 'fathers of A,' etc. Further, it would be difficult to give a definition of *one* or *two* which is not a statement in which one or two appears: he does not attempt to say that the other constituents of this 'statement' have not been defined, or that the definiendum is not unique. These could be his only grounds for attacking a definition, which is merely a statement in symbolic form of cases in which the number one or the number two appears. Also it seems a pity (line 9, p. 407) that he should fall into the error that he deplores in mathematical logicians, viz., the error of introducing the notion of truth (and truth value) when 'in no place.... they are defined.'

"I suppose that it was 'in the intoxication of the moment' that Professor Shaw called a propositional function of two variables a relation (p. 404, line 14), and let out of the bag the existence of a difference—hitherto, apparently, kept dark by mathematicians—between the properties of the roots of a quadratic equation and the properties of quadratic functions of *x*.

"Professor Shaw makes some strange remarks on page 413. If the collection of all integers does not exist (line 8) it seems hardly necessary to refute the proposition that it is possible to correlate the collection of all integers to some other infinite collections.

"It seems rather unsportsmanlike to rely upon people's short memories and call Poincaré's attacks on logistic successful. Might it not be well to remind people of the conclusions to which M. Couturat came at the end of his article in *The Monist* for October, 1912: 'Admitting the principles and primitive ideas of the logisticians, M. Poincaré has maintained that, setting out from these data, they cannot build up mathematics without another postulate—an appeal to intuition or a synthetic *a priori* judgment; and he has thought that he has discovered in their *logical* construction certain paralogisms (beggings of the question or vicious circles). I believe that I can conclude from the above discussion that not one of these theses is proved, and that, in particular, the logisticians have not committed any of the logical errors that are so lightly imputed to them.'"

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THE CAL-DIF-FLUK SAGA.

EDITORIAL INTRODUCTION.

Mr. J. M. Child has given, in the following "Saga," an amusing description of the results he has arrived at in his book on Barrow, just published in the series of "Open Court Classics." The closing lines represent the opinion he has formed from a consideration of the manuscripts of Leibniz, an annotated translation of which has been appearing in current numbers of *The Monist*, beginning with October, 1916, and continued in the April number and the present one.

The saga evidently refers to the question of the invention of the infinitesimal calculus. Isa-Roba is Barrow, Isa-Tonu is Newton, Zin-Bli is Leibniz, while Cavalieri is mentioned under the name of Ler-a-Cav. Gen-Tan-Agg stands for Barrow's *Gen*-eral method of *Tan*-gents and of *Agg*-regates; while Shun-Fluk and Cal-Dif ob-